QUANTUM FIELD THEORY

Tutorials (n'3)

1. **Complex field.-** Let us study the Hamiltonian of the free complex field.

   (a) Show that the operators $\hat{a}_p^{(\dagger)}$ and $\hat{\tilde{a}}_p^{(\dagger)}$ satisfy the canonical commutation relations as well as $[\hat{a}_p^{(\dagger)}, \hat{\tilde{a}}_p^{(\dagger)}] = 0$, where the annihilation operator $\hat{\tilde{a}}_p$ is associated to the anti-particle of 4-momentum p^μ.

   (b) Express the Hamiltonian H in terms of the annihilation (creation) operators $\hat{a}_p^{(\dagger)}$ and $\hat{\tilde{a}}_p^{(\dagger)}$.

2. **Propagator.-** Demonstrate that the propagator for the complex field operator $\hat{\phi}(x^\mu)$ obeys the property
$$iG(x^\mu - x'^\mu) =< 0|\tau[\hat{\phi}(x^\mu)\hat{\phi}^\dagger(x'^\mu)]|0 >= iG(x'^\mu - x^\mu)\,,$$
   where x^μ denotes the 4-coordinates and τ selects the time-ordering.

3. **Evolution operator.-** Check that the evolution operator
$$\hat{U}_0(t) = e^{-i\hat{H}_0 t}$$
   ($\hat{H}_0$ being the free Hamiltonian) for the free quantum state is unitary.

4. ***Wick* contraction.-** In this exercise we will relate the time-ordering, the normal-ordering and the *Wick* contraction.

   (a) Demonstrate the commutation relation, $[\hat{\phi}(x^\mu), \hat{\phi}(x'^\mu)] = 0$, among field operators involving identical time-components $t = t'$.

   (b) Demonstrate the equality property, $:\hat{\phi}(x^\mu)\hat{\phi}(x'^\mu): = :\hat{\phi}(x'^\mu)\hat{\phi}(x^\mu):$, where $:\hat{\phi}(x^\mu)\hat{\phi}(x'^\mu):$ denotes the normal-ordering.

   (c) Using previous question, show that the time-ordering between two fields is given by
$$\tau[\hat{\phi}_1(x^\mu)\hat{\phi}_2(x'^\mu)] = :\hat{\phi}_1(x^\mu)\hat{\phi}_2(x'^\mu): + \underbrace{\hat{\phi}_1(x^\mu)\hat{\phi}}_{}{}_2(x'^\mu)\,,$$
   where $\underbrace{\hat{\phi}_1(x^\mu)\hat{\phi}}_{}{}_2(x'^\mu)$ denotes the *Wick* contraction.